LESSON 4.1a

Evaluating Polynomial Functions

Today you will:

- Identify polynomial functions
- Practice using English to describe math processes and equations

Core Vocabulary:

- polynomial, p. 158
- polynomial function, p. 158
- end behavior, p. 159

Previous:

- monomial
- linear function
- quadratic function

What is a MONOMIAL?

- …mono means **…** “one”
- a number, a variable, or the product of a number and one or more variables **with whole number exponents**
- examples: $3, 3^2, x^2, 2x^3$
- the key is they can only have whole number exponents
- counter examples (not monomials): $3^{-2}, x^x, \sqrt{x}$ … each has an exponent that is not a whole number

What is a POLYNOMIAL?

- …poly means **…** “many” … one or more
- a monomial or a sum of monomials.

What is a POLYNOMIAL FUNCTION?

- a function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$
- examples: $f(x) = -2x^4 + 5x^3 - 3x^2 + x - 7$

  $g(x) = 3x^5 + 2x^2 - 4$

  $h(x) = 2x + 1$

Anatomy of a polynomial function

- $f(x) = a_n\, x^n + a_{n-1}x^{n-1} + \ldots + a_1\, x + a_0$ (general form)
- $f(x) = -3x^5 + 2x^4 - 5x^3 - x^2 + 4x - 7$ (example)

- Standard form:
  - Arranged with exponents in descending order from left to right

- Leading term:
  - The term with the highest exponent … example above, the leading term is $-3x^5$

- Leading coefficient:
  - The coefficient of the leading term … above, the leading coefficient is -3

- Degree of the polynomial
  - The value of the highest exponent
  - $f(x) = -2x^4 + 5x^3 - 3x^2 + x - 7$ … degree 4
  - $g(x) = 3x^5 + 2x^2 - 4$ … degree 5
  - $h(x) = 2x + 1$ … degree 1
  - $k(x) = 6$ … degree 0 (because it is actually $k(x) = 6x^0$)

Common Polynomial Functions			
Degree	**Type**	**Standard Form**	**Example**
0	Constant	$f(x) = a_0$	$f(x) = -14$
1	Linear	$f(x) = a_1x + a_0$	$f(x) = 5x - 7$
2	Quadratic	$f(x) = a_2x^2 + a_1x + a_0$	$f(x) = 2x^2 + x - 9$
3	Cubic	$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$	$f(x) = x^3 - x^2 + 3x$
4	Quartic	$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$	$f(x) = x^4 + 2x - 1$

Decide whether each function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

a. $f(x) = -2x^3 + 5x + 8$

b. $g(x) = -0.8x^3 + \sqrt{2}x^4 - 12$

c. $h(x) = -x^2 + 7x^{-1} + 4x$

d. $k(x) = x^2 + 3^x$

SOLUTION

a. The function **is a polynomial function** that is already written in standard form. It has degree 3 (cubic) and a leading coefficient of -2.

b. The function **is a polynomial function** written as $g(x) = \sqrt{2}x^4 - 0.8x^3 - 12$ in standard form. It has degree 4 (quartic) and a leading coefficient of $\sqrt{2}$.

c. The function is **not a polynomial** function because the term $7x^{-1}$ has an exponent that is not a whole number.

d. The function is **not a polynomial function** because the term 3^x does not have a variable base and an exponent that is a whole number.

Evaluate $f(x) = 2x^4 - 8x^2 + 5x - 7$ when $x = 3$.

SOLUTION

$f(x) = 2x^4 - 8x^2 + 5x - 7$ Write original equation.

$f(3) = 2(3)^4 - 8(3)^2 + 5(3) - 7$ Substitute 3 for *x*.

$= 162 - 72 + 15 - 7$ Evaluate powers and multiply.

$= 98$ Simplify.

End behavior of a polynomial's graph

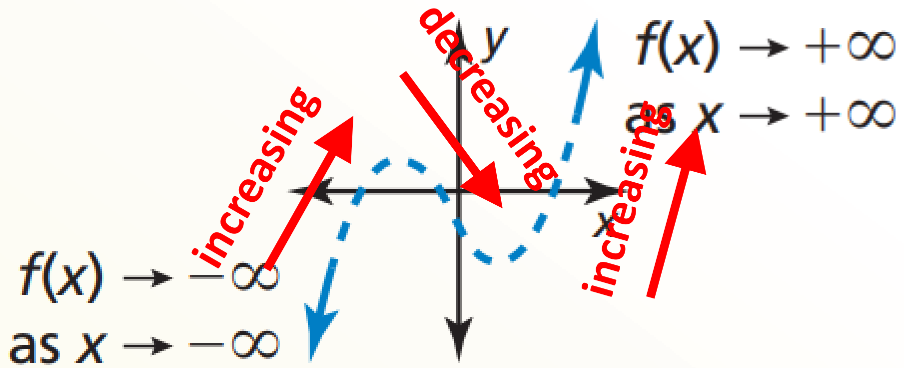
The behavior of the graph as x approaches positive infinity ($+\infty$) or negative infinity ($-\infty$).

Determined by:

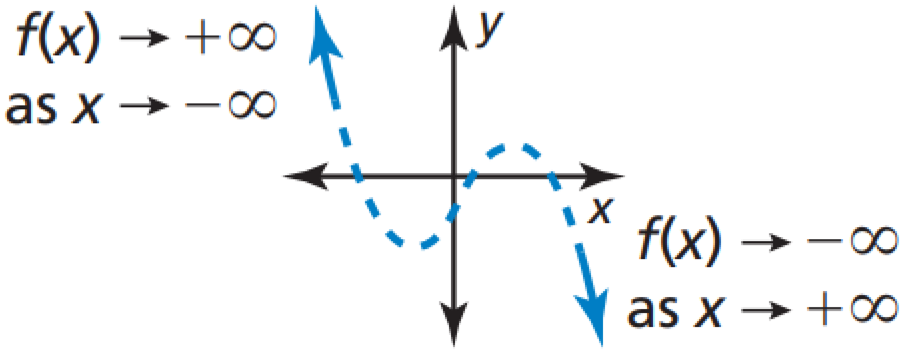
- the function's degree
- and the sign of its leading coefficient.

End Behavior of Polynomial Functions

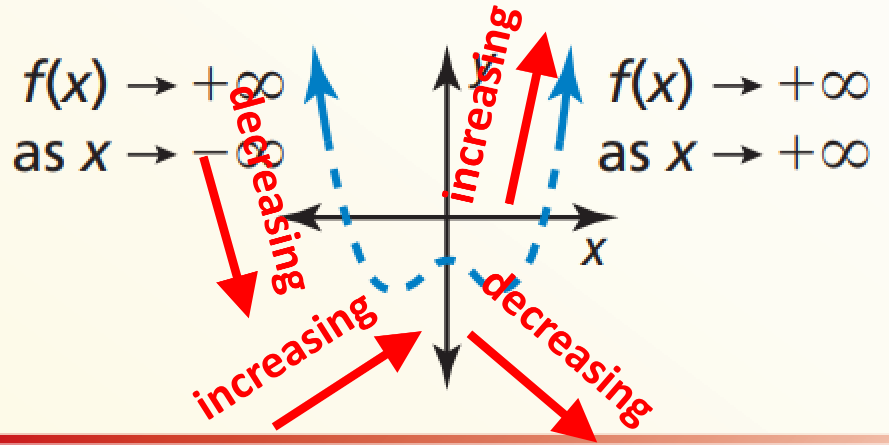
Degree: odd
Leading coefficient: positive



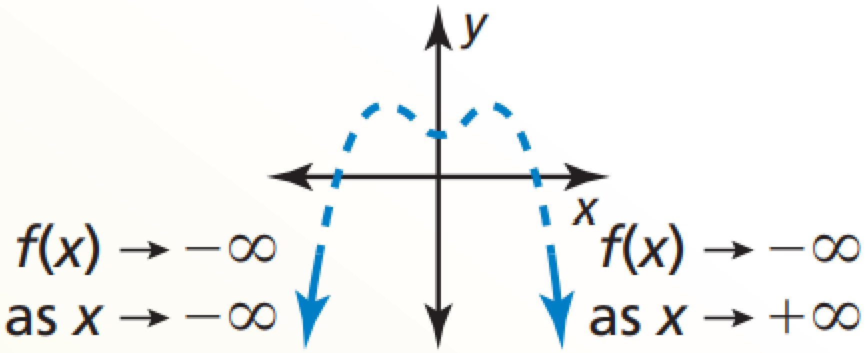
Degree: odd
Leading coefficient: negative



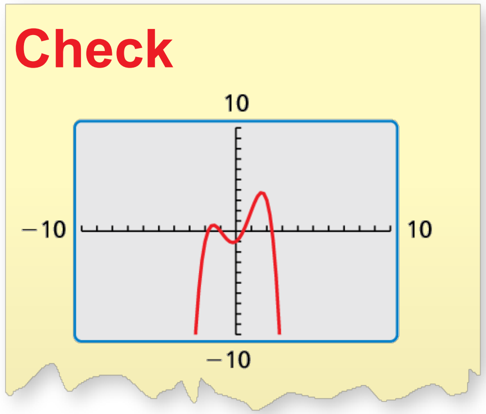
Degree: even
Leading coefficient: positive



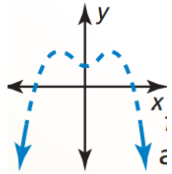
Degree: even
Leading coefficient: negative



Describe the end behavior of the graph of $f(x) = -0.5x^4 + 2.5x^2 + x - 1$.



SOLUTION



The function has degree 4 and leading coefficient −0.5.

Because the degree is even, we know both ends go the same direction.

Because the leading coefficient is negative, we know the graph opens down.

So as $x \to -\infty$, $f(x) \to -\infty$
And as $x \to +\infty$, $f(x) \to -\infty$

Check this by graphing the function on a graphing calculator, as shown.

$\leftarrow x$ $\quad$ $x \rightarrow$

$f(x)\downarrow$ $\quad$ $f(x)\downarrow$

Homework

Pg 162, #1-22